Base case: we can verify the summation formula for n = 1: 12 = 12. This is true.

Inductive step: let us assume we have proved the summation for some arbitrary k, i.e.

.

Now we consider the left-hand-side to find the next term in the sequence,

=

Add the next term of the sequence to the right-hand-side

+ 3() = = = which verifies that if the formula holds for one n, then it also holds for the next, completing the proof by induction.

1. **Proof by induction:** Base Case: for n = 1, = 1 which by the given recursive definition is true. Inductive step: Suppose that we have already proved that the statement is true for some arbitrary , i.e. . The goal is now to show that that the statement is true for n = k + 1. We can do this by substituting our base case:

. Which verifies that if the formula holds for one n, then it also holds for the next, completing the proof by induction.

1. **Proof by induction:** Base case: We can verify the base case n = 1, A(1, 1) = = 2 is true.

Inductive step: Let us assume we have proved the Ackermann function for some arbitrary n = k , i.e. A(1, k). The goal is now to show that statement is true for n = k + 1 by plugging in k + 1 for n. A(1, k + 1) = A (1 – 1, A(1, k + 1 – 1)) = A(0, A(1,k)) = A(1, . Therefore, A(1, k + 1) = 2 = . By induction we can conclude that A(1, n) = .

1. (a)

S = {1, 2, 3, 5, …}

for all

i.e

,

(b)

S = {1, 2, 4, 8, 16, …}

for all

i.e.

,

1. Using the division algorithm, we can verify that any positive integer divided by 10 that has a remainder of 7 has a last decimal digit 7. We can generalize this statement: if for an arbitrary positive integer x, (10x + 7)mod(10) = 7, then the values last digit is 7. We can also prove this by laws of modulo: (10x + 7)mod(10) = ((10x)mod(10) + (7)mod(10))mod(10) = (0 + 7)mod(10) = 7.

Next, we can prove that the base case, follows the guidelines as said above:

For x = 0, (10x + 7)mod(10) = 7.

Finally we can prove our other cases:

* 1. for x = 7 = 2(14) + 3 = 17.

with x = 1

17 S

* 1. = 49 + 8 = 57

with x = 5

S

Therefore, by structural induction, if a value is in S, then its last digit is 7.

1. For i:

2x + 3 = 27

x = 12.

The last digit of 12 is not 7, therefore not in S

For ii:

Again pick 27 therefore

is not in S.

Therefore, 27 is not in S.

1. Not sure how to start this one…
2. n/a
3. EXTRA CREDIT:

Text

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